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Fano hypersurfaces and Calabi-Yau supermanifolds

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ABSTRACT: In this paper, we study the geometrical interpretations associated with Sethi's proposed general correspondence between $\mathcal{N}=2$ Landau-Ginzburg orbifolds with integral $\hat{c}$ and $\mathcal{N}=2$ nonlinear sigma models. We focus on the supervarieties associated with $\hat{c}=3$ Gepner models. In the process, we test a conjecture regarding the superdimension of the singular locus of these supervarieties. The supervarieties are defined by a hypersurface $\widetilde{W}=0$ in a weighted superprojective space and have vanishing super-first Chern class. Here, $\widetilde{W}$ is the modified superpotential obtained by adding as necessary to the Gepner superpotential a boson mass term and/or fermion bilinears so that the superdimension of the supervariety is equal to $\hat{c}$. When Sethi's proposal calls for adding fermion bilinears, setting the bosonic part of $\widetilde{W}$ (denoted by $\widetilde{W}_{bos}$) equal to zero defines a Fano hypersurface embedded in a weighted projective space. In this case, if the Newton polytope of $\widetilde{W}_{bos}$ admits a nef partition, then the Landau-Ginzburg orbifold can be given a geometrical interpretation as a nonlinear sigma model on a complete intersection Calabi-Yau manifold. The complete intersection Calabi-Yau manifold should be equivalent to the Calabi-Yau supermanifold prescribed by Sethi's proposal.

KEYWORDS: Conformal Field Models in String Theory, String Duality, Sigma Models

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Contents

1 Introduction

Mirror symmetry [1] is a duality between string theories propagating on distinct but mirror target spaces. Consider a string theory compactified on a Calabi-Yau manifold X which is related by mirror symmetry to a string theory compactified on a Calabi-Yau manifold Y. The mirror map relates the Hodge numbers of X and Y by $h_X^{p,q} = h_Y^{\mathcal{D}-p,q}$, where $\mathcal{D}$ is the complex dimension of X and Y. Thus, the mirror map identifies the complex structure moduli space of X with the Kähler moduli space of Y and vice-versa.

A *rigid* Calabi-Yau manifold has no complex structure moduli. The mirror of such a manifold has no Kähler moduli and hence cannot be a Kähler manifold in the conventional sense. Thus, Calabi-Yau manifolds cannot be the most general geometrical framework for understanding mirror symmetry. The first progress towards generalizing this framework came when Schimmrigk [2] suggested that higher-dimensional Fano varieties could provide mirrors for rigid Calabi-Yau manifolds. The name "generalized Calabi-Yau" was introduced by Candelas et al. [3] for these mirror manifolds. Later progress came when Sethi [4] proposed a general correspondence between $\mathcal{N}=2$ Landau-Ginzburg orbifolds with integral $\hat{c} \equiv c/3$ (where c is the central charge) and $\mathcal{N}=2$ nonlinear sigma models. Here, the target space of the nonlinear sigma model is either a Calabi-Yau manifold or a Calabi-Yau *supermanifold*. Using this proposal, Sethi argued that the mirror of a rigid Calabi-Yau manifold is a Calabi-Yau supermanifold and hence mirror symmetry should be viewed as a relation among Calabi-Yau manifolds and Calabi-Yau supermanifolds alike. The bodies of the supermanifolds are the Fano varieties mentioned above. Witten [5] described $\mathcal{N}=2$ nonlinear sigma models on Calabi-Yau manifolds and $\mathcal{N}=2$ Landau-Ginzburg orbifolds as being different phases of $\mathcal{N}=2$ gauged linear sigma models. Sethi's work and [6] have led others [7] to study $\mathcal{N}=2$ gauged linear sigma models which have a phase described by an $\mathcal{N}=2$ nonlinear sigma model on a Calabi-Yau supermanifold. The gauged linear

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sigma model framework allows the argument establishing a correspondence between the nonlinear sigma model and the Landau-Ginzburg orbifold to be made more robust.

A Calabi-Yau supermanifold M obtained from Sethi's proposed correspondence would be realized by resolving the singularities of a supervariety $\mathcal{M}$ embedded in a weighted superprojective space

$$\mathbf{WSP}^{(n|2m)} \equiv \mathbf{WSP}(n_{z_1}, \ldots, n_{z_{n+1}} | n_{\eta_1}, \ldots, n_{\eta_{2m}}) \,. \tag{1.1}$$

Here, $\mathcal{M}$ is defined by the zero locus of a transverse,[1] quasihomogeneous superpotential $\widetilde{W} = \widetilde{W}(z_\mu; \eta_\alpha)$, where

$$z_\mu \simeq \lambda^{n_{z_\mu}} z_\mu \ (\mu = 1, \ldots, n+1) \,, \quad \eta_\alpha \simeq \lambda^{n_{\eta_\alpha}} \eta_\alpha \ (\alpha = 1, \ldots, 2m) \tag{1.2}$$

are homogeneous bosonic and fermionic coordinates of weights n_{z_μ} and n_{η_α}, respectively. Since $\widetilde{W}$ is quasihomogeneous, it satisfies

$$\widetilde{W}(\lambda^{n_{z_\mu}} z_\mu; \lambda^{n_{\eta_\alpha}} \eta_\alpha) = \lambda^d \, \widetilde{W}(z_\mu; \eta_\alpha) \,, \tag{1.3}$$

where d is the degree of quasihomogeneity. The superpotential $\widetilde{W}$ is obtained from the transverse, quasihomogeneous superpotential $W = W(\Phi_a)$ of an $\mathcal{N} = 2$ Landau-Ginzburg model with integral $\hat{c}$ by truncating each chiral superfield Φ_a $(a = 1, \ldots, N)$ to its lowest bosonic component ϕ_a, setting $\phi_a = z_a$, and then adding boson mass terms $z_{N+1}^2 + \cdots + z_{n+1}^2$ and/or fermion bilinears $\eta_1\eta_2 + \cdots + \eta_{2m-1}\eta_{2m}$ to W so that

$$\widetilde{\mathcal{D}} \equiv (n+1) - 2m - 2 = \hat{c} \,. \tag{1.4}$$

Here, $\widetilde{\mathcal{D}}$ is the *superdimension* of the Calabi-Yau supermanifold. The condition (1.4) allows a change of variables with constant Jacobian to be made such that one of the new variables appears only linearly in the modified superpotential $\widetilde{W}$. The change of variables is not one-to-one, so the modified theory must be orbifolded by the diagonal subgroup of its phase symmetries. Integrating the linear new variable out of the path integral for the modified action as a Lagrange multiplier yields a super-delta function constraint which corresponds to $\mathcal{M}$ having vanishing super-first Chern class.

When $m \neq 0$, setting the bosonic part of $\widetilde{W}$ (denoted by $\widetilde{W}_{bos}$) equal to zero defines a *Fano hypersurface* $\mathcal{F}$ embedded in a weighted projective space

$$\mathbf{WP}^n \equiv \mathbf{WP}(n_{z_1}, \ldots, n_{z_{n+1}}) \,. \tag{1.5}$$

In this case, if the Newton polytope Δ corresponding to $\widetilde{W}_{bos}$ admits a *nef partition* $\Delta = \Delta_1 + \cdots + \Delta_r$, then the Landau-Ginzburg orbifold can be given a geometrical interpretation as a nonlinear sigma model on a complete intersection Calabi-Yau manifold defined by equations $f_i = 0$ $(i = 1, \ldots, r)$ [8]. Here, Δ_i is the Newton polytope of f_i. The complete intersection Calabi-Yau manifold should be equivalent (in the sense of [6]) to the Calabi-Yau supermanifold prescribed by Sethi's proposed correspondence. When $m = 0$, the

[1]Transverse $\widetilde{W}$ means that $\widetilde{W} = 0$ and $d\widetilde{W} = 0$ have no common solution except at the origin.

constraint $\widetilde{W} = 0$ defines a Calabi-Yau variety $\mathcal{X}$ embedded in $\mathbf{WP}^n$. Resolving $\mathcal{X}$ would yield a Calabi-Yau manifold X of complex dimension

$$\mathcal{D} \equiv (n+1) - 2 = \hat{c}\,. \tag{1.6}$$

The complex dimension of the singular locus of $\mathcal{X}$ satisfies [9]

$$0 \leq \dim(\mathrm{Sing}(\mathcal{X})) \leq \mathcal{D} - 2\,, \quad \mathrm{Sing}(\mathcal{X}) = \mathcal{X} \cap \mathrm{Sing}(\mathbf{WP}^n)\,. \tag{1.7}$$

To obtain a Calabi-Yau supermanifold M by resolving the singularities of $\mathcal{M}$, one might infer from the discussion in [4] that the superdimension of the singular locus of $\mathcal{M}$ must satisfy

$$\mathrm{sdim}(\mathrm{Sing}(\mathcal{M})) \leq \widetilde{\mathcal{D}} - 1\,, \quad \mathrm{Sing}(\mathcal{M}) = \mathcal{M} \cap \mathrm{Sing}\left(\mathbf{WSP}^{(n|2m)}\right)\,. \tag{1.8}$$

The result (1.7) and the equivalence discussed in [6] suggests the following stronger conjecture:

Conjecture 1.1 *To obtain a Calabi-Yau supermanifold M by resolving the singularities of $\mathcal{M}$, the superdimension of the singular locus of $\mathcal{M}$ must satisfy*

$$\mathrm{sdim}(\mathrm{Sing}(\mathcal{M})) \leq \widetilde{\mathcal{D}} - 2\,, \quad \mathrm{Sing}(\mathcal{M}) = \mathcal{M} \cap \mathrm{Sing}\left(\mathbf{WSP}^{(n|2m)}\right)\,.$$

In this paper, we will test the above conjecture for $\widetilde{\mathcal{D}} = 3$. This will be achieved by studying the geometrical interpretations prescribed by Sethi's proposed correspondence for *Gepner models* [10] with $\hat{c} = 3$. Since $\widetilde{W}$ is quasihomogeneous of degree d, the weights of the fermions in each fermion bilinear $\eta_{2k-1}\eta_{2k}$ $(k = 1, \ldots, m)$ must satisfy

$$n_{\eta_{2k-1}} + n_{\eta_{2k}} = d\,. \tag{1.9}$$

Requiring either the singular locus constraint (1.8) or conjecture 1.1 to hold further restricts the fermionic weights. We have written a computer program which allows these restrictions to be implemented. In principle, one could determine the fermionic weights by requiring agreement between the Hodge diamond of the Landau-Ginzburg orbifold and the Hodge diamond of $\mathcal{M}$. The former Hodge diamond can be computed using the techniques of [11] whereas insight into the structure of the latter Hodge diamond can be obtained from the heuristic approach of [4] based on orbifold considerations [12]. We will compare the fermionic weights obtained in this way with those obtained from our computer program.

This paper is organized as follows: In section 2, we discuss the application of Sethi's proposed Landau-Ginzburg orbifold/nonlinear sigma model correspondence to Gepner models. In section 3, we describe how the singular locus constraint (1.8) restricts the fermionic weights and work through an example. It is a trivial step to replace (1.8) with conjecture 1.1 in our computer program. The analysis of section 4 compares the fermionic weights obtained from our computer program with those obtained from the cohomological approach described in the previous paragraph. Several examples are included which highlight the similarities and differences. Concluding remarks are given in section 5. Finally, in the appendix, we tabulate the families of hypersurfaces associated with our supervarieties.

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Here, the fermionic weights are determined with our computer program by requiring (1.8) and (1.9) to be satisfied. For each hypersurface family, the Hodge numbers $h^{1,1}$ and $h^{2,1}$ and the Euler number of the associated Landau-Ginzburg orbifold are given. When the Newton polytope corresponding to $\widetilde{W}_{bos}$ admits a nef partition, this is indicated. We also indicate when the Newton polytope of $\widetilde{W}_{bos}$ is nonreflexive Gorenstein.

2 Gepner/NLSM correspondence

The worldsheet action for an $\mathcal{N}=2$ Landau-Ginzburg model is [13]

$$S=\int d^2\mathbf{z}\, d^4\theta\, K\left(\Phi_a,\overline{\Phi}_a\right)+\left(\int d^2\mathbf{z}\, d^2\theta\, W(\Phi_a)+c.c.\right), \tag{2.1}$$

Here, the integral involving the Kähler potential K is called the D-term, the integral involving the superpotential W is called the F-term, and Φ_a $(a=1,\ldots,N)$ are chiral superfields. The D-term contains only irrelevant operators whereas the F-term contains relevant operators. Thus, the superpotential defines a universality class under renormalization group flow. Requiring the superpotential to be transverse and quasihomogeneous is believed to ensure the existence of a unique, nontrivial IR fixed point which is conformally invariant. At this fixed point, the action (2.1) provides a Lagrangian description of an $\mathcal{N}=2$ minimal model [13–15] with

$$\hat{c}=2\sum_{a=1}^{N}\left(\frac{1}{2}-q_{\Phi_a}\right), \quad q_{\Phi_a}\equiv n_{\Phi_a}/d\,. \tag{2.2}$$

The 10,839 transverse, quasihomogeneous Landau-Ginzburg superpotentials corresponding to $\mathcal{N}=2$ superconformal theories with $\hat{c}=3$ were classified in [16]. A subset of these correspond to Gepner models with $\hat{c}=3$. A Gepner model [10] is a string model constructed as an orbifold of a tensor product of $\mathcal{N}=2$ minimal models. The central charge of the ith $\mathcal{N}=2$ minimal model in the tensor product is

$$c_i=\frac{3k_i}{k_i+2}\quad (k_i=1,2,\ldots)\,, \tag{2.3}$$

where k_i is the *level* of the $\mathcal{N}=2$ superconformal algebra [17]. To obtain an anomaly-free compactification to D spacetime dimensions ($D<10$, even), the internal contribution to the central charge must be

$$c=\sum_i c_i=\frac{3}{2}(10-D)\,. \tag{2.4}$$

The work of [18, 19] associated Calabi-Yau manifolds to a large class of Gepner models. Sethi's proposal [4] prescribes a geometrical interpretation for *all* Gepner models. This prescription is as follows:

1. Start by associating superpotential terms [14]

   - $W_i=x_i^{k_i+2}$ to A-models of level k_i,
   - $W_i=x_i^{\frac{k_i}{2}+1}+x_iy_i^2$ to D-models of level k_i (even),

- $W_i = x_i^3 + y_i^4$, $W_i = x_i^3 + x_i y_i^3$, and $W_i = x_i^3 + y_i^5$ to E_6-, E_7-, and E_8-models of level $k_i = 10, 16, 28$, respectively.

The tensor product of r subtheories yields a transverse, quasihomogeneous Gepner superpotential

$$W = \sum_{i=1}^{r} W_i = \sum_{i=1}^{r} \left(x_i^{l_{x_i}} + x_i^{\bar{l}_{x_i}} y_i^{l_{y_i}} \right). \tag{2.5}$$

For A-models, $y_i = 0$ and $l_{x_i} = k_i + 2$. For D-models, $l_{x_i} = k_i/2 + 1$, $\bar{l}_{x_i} = 1$, and $l_{y_i} = 2$. For E_6-, E_7-, and E_8-models, $l_{x_i} = 3, 3, 3$, $\bar{l}_{x_i} = 0, 1, 0$, and $l_{y_i} = 4, 3, 5$, respectively. In all cases,

$$n_{x_i} l_{x_i} = d\,. \tag{2.6}$$

Additionally, for D- and E-models,

$$n_{x_i} \bar{l}_{x_i} + n_{y_i} l_{y_i} = d\,. \tag{2.7}$$

The x_i and nonzero y_i $(i = 1, \ldots, r)$ are identified with the z_a $(a = 1, \ldots, N)$ described in the introduction according to the convention $z_1 = x_1$,

$$z_2 = \begin{cases} y_1 & (y_1 \neq 0) \\ x_2 & (y_1 = 0)\,, \end{cases}$$

and so on.

2. Add as necessary to W a single[2] boson mass term z^2 and/or fermion bilinears $\eta_1\eta_2 + \cdots + \eta_{2m-1}\eta_{2m}$ so that

$$(n + 1) - 2m - 2 = \hat{c}\,. \tag{2.8}$$

We thus obtain the modified superpotential

$$\widetilde{W} = \begin{cases} W & (\hat{c} = N - 2) \\ W + z^2 & (\hat{c} > N - 2) \\ W + \sum_{k=1}^{m} \eta_{2k-1}\eta_{2k} & (\hat{c} < N - 2,\ \sum_{a=1}^{N} q_{z_a} \text{integral}) \\ W + z^2 + \sum_{k=1}^{m} \eta_{2k-1}\eta_{2k} & (\hat{c} < N - 2,\ \sum_{a=1}^{N} q_{z_a} \text{half-integral})\,. \end{cases} \tag{2.9}$$

The added fields have no effect on the chiral ring or the conformal fixed point to which the theory flows. The condition (2.8) allows a change of variables with constant Jacobian to

[2]There are three possibilities for an arbitrary Landau-Ginzburg model with integral $\hat{c}$:

- For $\hat{c} = N - 2$, no fields need to be added.
- For $\hat{c} > N - 2$, boson mass terms $z_{N+1}^2 + \cdots + z_{n+1}^2$ are required.
- For $\hat{c} < N - 2$, the condition that $\hat{c}$ be integral implies that the sum of the charges $\sum_{a=1}^{N} q_{z_a}$ can be either integral or half-integral. The former requires adding fermion bilinears $\eta_1\eta_2 + \cdots + \eta_{2m-1}\eta_{2m}$ whereas the latter requires adding a single boson mass term z_{n+1}^2 and fermion bilinears $\eta_1\eta_2 + \cdots + \eta_{2m-1}\eta_{2m}$.

For Gepner models, $\hat{c}$ is never greater than $N - 2$ by more than 1. Thus, it is never necessary to add more than one boson mass term to the Gepner superpotential.

be made such that one of the new variables appears only linearly in the modified superpotential. The change of variables is not one-to-one, so the modified theory must orbifolded by the diagonal subgroup of its phase symmetries. When $m = 0$ ($m \neq 0$), integrating the linear new variable out of the path integral for the modified action as a Lagrange multiplier yields a (super-)delta function constraint which corresponds to the bosonic variety $\mathcal{X}$ (supervariety $\mathcal{M}$) defined by $\widetilde{W} = 0$ having vanishing (super-)first Chern class. The first Chern class of $\mathcal{X}$ vanishes when

$$\sum_{\mu=1}^{n+1} n_{z_\mu} - d = 0\,, \tag{2.10}$$

whereas the super-first Chern class of $\mathcal{M}$ vanishes when

$$\sum_{\mu=1}^{n+1} n_{z_\mu} - \sum_{\alpha=1}^{2m} n_{\eta_\alpha} - d = 0\,. \tag{2.11}$$

3 Fermionic weights and the singular locus constraint

As described in the introduction, we require the fermionic weights to be consistent with the singular locus constraint (1.8) and the quasihomogeneity constraint (1.9). The restriction placed on the fermionic weights by the latter constraint is obvious. Let us now explain what consistency with the former constraint means.

The supervariety $\mathcal{M}$ defined by the hypersurface $\widetilde{W} = 0$ has a $\mathbf{Z}_p$ fixed point set under the weighted projective identification (1.2) if and only if the following conditions are both satisfied:

1. The index set $B_p \equiv \{\mu | \lambda_p^{n_{z_\mu}} = 1\}$ is nonempty, where $\lambda_p \equiv e^{2\pi i/p}$.

2. The quantity

$$\mathcal{D}_p = \begin{cases} |B_p| - 2 & (d/p \in \mathbf{Z}) \\ |B_p| - 1 & (d/p \notin \mathbf{Z})\,. \end{cases} \tag{3.1}$$

   satisfies $\mathcal{D}_p \geq 0$ if the index set $F_p \equiv \{\alpha | \lambda_p^{n_{\eta_\alpha}} = 1\}$ is empty.

There are no purely fermionic $\mathbf{Z}_p$ fixed point sets because the body of $\mathcal{M}$ is embedded in $\mathbf{WP}^n$. The superdimension of the $\mathbf{Z}_p$ fixed point sets which do exist is given by

$$\widetilde{\mathcal{D}}_p = \begin{cases} |B_p| - |F_p| - 2 & (d/p \in \mathbf{Z}) \\ |B_p| - |F_p| - 1 & (d/p \notin \mathbf{Z})\,. \end{cases} \tag{3.2}$$

In the case $d/p \in \mathbf{Z}$, the -2 arises because the weighted projective identification and the hypersurface equation each reduce the superdimension by 1. In the case $d/p \notin \mathbf{Z}$, the hypersurface equation is identically satisfied and hence does not reduce the superdimension. Consistency with the singular locus constraint (1.8) means that

$$\widetilde{\mathcal{D}}_p \leq \hat{c} - 1 \quad \forall p\,, \tag{3.3}$$

where we have used (1.4). We see from (3.2) that $\mathbf{Z}_p$ fixed point sets with $|B_p| \leq |F_p|$ have negative superdimension and hence are consistent with (1.8). Thus, when checking for consistency with (1.8), we can focus our attention on $\mathbf{Z}_p$ fixed point sets with $|B_p| > |F_p|$. Note that the bosonic part of such a $\mathbf{Z}_p$ fixed point set has complex dimension given by (3.1).

To illustrate the above, let us consider a concrete example.

Example 3.1 *Consider a Gepner model with level/invariant structure* 10_D 10_D 1_A 1_A 1_A 1_A*. Following the procedure described in section 2, we obtain the quasihomogeneous degree* $d = 12$ *modified superpotential*

$$\widetilde{W} = \sum_{i=1}^{2} (x_i^6 + x_i y_i^2) + \sum_{i=3}^{6} x_i^3 + z^2 + \eta_1\eta_2 + \eta_3\eta_4 \,.$$

The hypersurface $\widetilde{W} = 0$ *defines a supervariety* $\mathcal{M}$ *embedded in*

$$\mathbf{WSP}(2,5,2,5,4,4,4,4,6|n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4}) \,.$$

We will denote the family of quasihomogeneous degree $d = 12$ *hypersurfaces embedded in this weighted superprojective space by*

$$\mathbf{WSP}(2,5,2,5,4,4,4,4,6|n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4})[12] \,.$$

A hypersurface in this family would also be obtained from a Gepner model with level and invariant structure 4_D 4_D 10_D 10_D *or* 4_D 10_D 10_D 1_A 1_A*.*

According to (1.9), the fermions in each bilinear can take on the values (modulo a relabelling of the fermions)

$$\left(n_{\eta_{2k-1}}, n_{\eta_{2k}}\right) \in \{(1,11),(2,10),(3,9),(4,8),(5,7),(6,6)\} \,. \tag{3.4}$$

To further constrain the fermionic weights, we now consider the $\mathbf{Z}_p$ $(p = 2,4,5)$ *fixed point sets. First, we determine the complex dimension of the bosonic parts of these fixed point sets:*

$\underline{p = 2}$*: The bosonic part of the* $\mathbf{Z}_2$ *fixed point set is*

$$\sum_{i=1}^{2} (x_i^6 + x_i y_i^2) + \sum_{i=3}^{6} x_i^3 + z^2 = 0 \,, \quad y_1 = y_2 = 0 \,.$$

There are $|B_2| = 7$ *bosons* (x_i $(i = 1,\ldots,6)$ and z) *in this fixed point set. Since* $d/p = 6 \in \mathbf{Z}$*, (3.1) gives*

$$\mathcal{D}_2 = |B_2| - 2 = 5 \,. \tag{3.5}$$

$\underline{p = 4}$*: The bosonic part of the* $\mathbf{Z}_4$ *fixed point set is*

$$\sum_{i=3}^{6} x_i^3 = 0 \,, \quad x_i = y_i = 0 \quad (i = 1,2) \,, \quad z = 0 \,.$$

j	$(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4})_j$	$\widetilde{\mathcal{D}}_2^{(j)}$	$\widetilde{\mathcal{D}}_4^{(j)}$	$\widetilde{\mathcal{D}}_5^{(j)}$
1	(1,11,1,11)	5	2	1
2	(1,11,2,10)	3	2	0
3	(1,11,3,9)	5	2	1
4	(1,11,4,8)	3	0	1
5	(1,11,5,7)	5	2	0
6	(1,11,6,6)	3	2	1
7	(2,10,2,10)	1	2	-1
8	(2,10,3,9)	3	2	0
9	(2,10,4,8)	1	0	0
10	(2,10,5,7)	3	2	-1
11	(2,10,6,6)	1	2	0
12	(3,9,3,9)	5	2	1
13	(3,9,4,8)	3	0	1
14	(3,9,5,7)	5	2	0
15	(3,9,6,6)	3	2	1
16	(4,8,4,8)	1	-2	1
17	(4,8,5,7)	3	0	0
18	(4,8,6,6)	1	0	1
19	(5,7,5,7)	5	2	-1
20	(5,7,6,6)	3	2	0
21	(6,6,6,6)	1	2	1

Table 1. Computation of $\widetilde{\mathcal{D}}_p^{(j)} = \mathcal{D}_p - |F_p^{(j)}|$ $(p = 2, 4, 5)$.

There are $|B_4| = 4$ *bosons* (x_i $(i = 3, \ldots, 6)$) *in this fixed point set. Since* $d/p = 3 \in \mathbf{Z}$, *(3.1) gives*

$$\mathcal{D}_4 = |B_4| - 2 = 2\,. \tag{3.6}$$

$\underline{p = 5}$: *The bosonic part of the* $\mathbf{Z}_5$ *fixed point set is*

$$\sum_{i=1}^{2} \left(x_i^6 + x_i y_i^2\right) = 0\,, \quad x_i = 0 \quad (i = 1, \ldots, 6)\,, \quad z = 0\,.$$

There are $|B_5| = 2$ *bosons* (y_1 and y_2) *in this fixed point set. Since* $d/p = 12/5 \notin \mathbf{Z}$, *(3.1) gives*

$$\mathcal{D}_5 = |B_5| - 1 = 1\,. \tag{3.7}$$

Next, let $\widetilde{\mathcal{D}}_p^{(j)} = \mathcal{D}_p - |F_p^{(j)}|$ *be the superdimension of the* $\mathbf{Z}_p$ *fixed point set for the* j*th distinct (modulo relabelling) fermionic weight combination* $(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4})_j$ *consistent with (1.9). For* $m = 2$ *fermion bilinears, there are* $\frac{1}{2}\left[\frac{d}{2}\right]\left(\left[\frac{d}{2}\right]+1\right)$ *such combinations, where* $[x]$ *denotes the integer part of* x*. Thus, for this example,* $j = 1, \ldots, 21$*. The* $\widetilde{\mathcal{D}}_p^{(j)}$ *are given in table 1.*

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Finally, exclude all fermionic weight combinations which do not satisfy

$$\widetilde{\mathcal{D}}_p^{(j)} \leq \hat{c} - 1 = 2 \ \forall p \, .$$

This leaves the fermionic weight combinations

$$(2,10,2,10)\,,\ (2,10,4,8)\,,\ (2,10,6,6)\,,\ (4,8,4,8)\,,\ (4,8,6,6)\,,\ (6,6,6,6)\,.$$

We can use the parameters k and l defined in the appendix to express these combinations in the compact form $(2k, 12-2k, 2l, 12-2l)$. Here, $k = 1, \ldots, 3$ and $l = 1, \ldots, 3$. It is understood that repeated fermionic weight combinations generated with this notation are counted only once. In this notation, the hypersurface $\widetilde{W} = 0$ defines a supervariety $\mathcal{M}$ embedded in $\mathbf{WSP}(2,5,2,5,4,4,4,4,6|2k,12-2k,2l,12-2l)$ *and is a member of hypersurface family 250 of the appendix:*

$$\mathbf{WSP}(2,5,2,5,4,4,4,4,6|2k,12-2k,2l,12-2l)[12]\,.$$

4 Analysis

We have written a computer program which takes as input data which encodes the superpotential W for $\hat{c} = 3$ Gepner models. Next, the program determines the modified superpotential $\widetilde{W}$ as explained in section 2. When $\widetilde{W}$ depends on five bosonic fields and no fermionic fields, the output of the program is the hypersurface family

$$\mathbf{WP}(n_{z_1}, \ldots, n_{z_{n+1}})[d]$$

corresponding to the hypersurface $\widetilde{W} = 0$ which defines a bosonic variety $\mathcal{X}$ embedded in $\mathbf{WP}(n_{z_1}, \ldots, n_{z_{n+1}})$. For the remaining cases, the output is the hypersurface family

$$\mathbf{WSP}(n_{z_1}, \ldots, n_{z_{n+1}}|n_{\eta_1}, \ldots, n_{\eta_{2m}})[d]$$

corresponding to the hypersurface $\widetilde{W} = 0$ which defines a supervariety $\mathcal{M}$ embedded in $\mathbf{WSP}(n_{z_1}, \ldots, n_{z_{n+1}}|n_{\eta_1}, \ldots, n_{\eta_{2m}})$. The fermionic weights are determined by requiring the singular locus constraint (1.8) and the quasihomogeneity constraint (1.9) to be satisfied. In this manner, we obtain at least one fermionic weight solution for each $\hat{c} = 3$ Gepner model corresponding to a supervariety $\mathcal{M}$ through Sethi's proposed correspondence. The 254 hypersurface families associated with these supervarieties are tabulated in the appendix. When we replace the singular locus constraint (1.8) with conjecture 1.1, this results in models corresponding to the hypersurface families 50, 94, 95, 121, 125, and 229 of the appendix having no solution for the fermionic weights.

In principle, the singularities which do arise could be determined by identifying the fermionic weights which yield agreement between the Hodge diamond of the Landau-Ginzburg orbifold and the Hodge diamond of $\mathcal{M}$. The Landau-Ginzburg orbifold Hodge diamond can be computed using the techniques of [11]. Such calculations can be done quickly with the software package PALP [20]. Unfortunately, at present, there is no super-cohomology theory which allows the Hodge diamond of $\mathcal{M}$ to be computed.

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The Hodge diamond of $\mathcal{X}$ can be computed using the orbifold techniques of [12]. This is possible because, with a change of coordinates

$$z_\mu = (\zeta_\mu)^{n_{z_\mu}}\,, \tag{4.1}$$

the hypersurface $\mathbf{WP}(n_{z_1},\ldots,n_{z_{n+1}})[d]$ which defines $\mathcal{X}$ can be written as an orbifold of a hypersurface in a homogeneous projective space $\mathbf{P}^n$, i.e.

$$\mathbf{WP}(n_{z_1},\ldots,n_{z_{n+1}})[d] = \frac{\mathbf{P}^n[d]}{\mathbf{Z}_{n_{z_1}}\times\cdots\times\mathbf{Z}_{n_{z_{n+1}}}}\,. \tag{4.2}$$

In contrast, the hypersurface $\mathbf{WSP}(n_{z_1},\ldots,n_{z_{n+1}}|n_{\eta_1},\ldots,n_{\eta_{2m}})[d]$ which defines $\mathcal{M}$ cannot be written as an orbifold of a hypersurface in homogeneous superprojective space $\mathbf{SP}^{(n|2m)}$. This is because the analogue of (4.1) does not make sense for Grassmann coordinates. Nevertheless, as described in [4], we can use orbifold considerations to gain insight into the structure of the Hodge diamond of $\mathcal{M}$. In the following examples, we will use this heuristic reasoning and compare the resulting fermionic weights with those obtained from the singular locus constraint (1.8) and conjecture 1.1.

Example 4.1 *A hypersurface in the family* $\mathbf{WSP}(1,1,2,2,2,2,2|n_{\eta_1},n_{\eta_2})[6]$ *can be obtained from a Gepner model with any of the following level/invariant structures:*

$$4_D\ 4_D\ 4_A\ 4_A\ 1_A\,,\quad 4_D\ 4_A\ 4_A\ 1_A\ 1_A\ 1_A\,,\quad 4_A\ 4_A\ 1_A\ 1_A\ 1_A\ 1_A\ 1_A\,.$$

For definiteness, we will focus on the last of these. Following the procedure described in section 2, we obtain the modified superpotential

$$\widetilde{W} = x_1^6 + x_2^6 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + \eta_1\eta_2\,.$$

This example was discussed in [4]. Here, we simply note that cohomology considerations, conjecture 1.1, and, as indicated by hypersurface family 179 of the appendix, the singular locus constraint (1.8) all yield the same unique result $(n_{\eta_1},n_{\eta_2}) = (2,4)$.

Example 4.2 *A hypersurface in the family* $\mathbf{WSP}(1,1,4,4,4,4,6|n_{\eta_1},n_{\eta_2})[12]$ *can be obtained from a Gepner model with any of the following level/invariant structures:*

$$4_D\ 10_A\ 4_A\ 2_A\ 1_A\,,\quad 10_A\ 4_A\ 2_A\ 1_A\ 1_A\ 1_A\,,\quad 4_D\ 4_D\ 10_A\ 10_A\,,$$
$$4_D\ 10_A\ 10_A\ 1_A\ 1_A\,,\quad 10_A\ 10_A\ 1_A\ 1_A\ 1_A\ 1_A\,.$$

For definiteness, we will focus on the last of these. Following the procedure described in section 2, we obtain the modified superpotential

$$\widetilde{W} = x_1^{12} + x_2^{12} + x_3^3 + x_4^3 + x_5^3 + x_6^3 + z^2 + \eta_1\eta_2\,.$$

Employing the heuristic reasoning of [4], we find for $(n_{\eta_1},n_{\eta_2}) = (4,8)$ *that the Hodge diamond of* $\mathcal{M}$ *is*

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 7 & & 0 & \\
1 & & 79 & & 79 & & 1 \\
 & 0 & & 7 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 0 & & 0 & \\
1 & & 79 & & 79 & & 1 \\
 & 0 & & 0 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
+
\begin{array}{ccccccc}
 & & & 0 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 6 & & 0 & \\
0 & & 0 & & 0 & & 0 \\
 & 0 & & 6 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 0 & & &
\end{array}
+
\begin{array}{ccccccc}
 & & & 0 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 1 & & 0 & \\
0 & & 0 & & 0 & & 0 \\
 & 0 & & 1 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 0 & & &
\end{array}
. \tag{4.3}
$$

The first term on the right-hand side of (4.3) is the contribution arising from the untwisted sector. The second term includes the contribution of the fixed point set associated with the third twisted sector (the upper 6) and the fixed point set associated with the ninth twisted sector (the lower 6). Finally, the last term arises from the identity and volume forms of the fixed point set associated with the sixth twisted sector. Our result (4.3) agrees with the Hodge diamond of the associated Landau-Ginzburg orbifold. For all other fermionic weight assigments consistent with (1.9), we do not obtain this agreement. Thus, in this example, using these heuristic arguments, we obtain a unique result

$$(n_{\eta_1}, n_{\eta_2}) = (4, 8)\,.$$

We note that this result agrees with what would be obtained from conjecture 1.1. In contrast, as indicated by hypersurface family 10 of the appendix, the singular locus constraint (1.8) allows

$$(n_{\eta_1}, n_{\eta_2}) \in \{(2,10), (4,8), (6,6)\}\,.$$

Example 4.3 *A hypersurface in the family* $\mathbf{WSP}(1,3,3,3,4,4,6|n_{\eta_1}, n_{\eta_2})[12]$ *can be obtained from a Gepner model with either of the following level/invariant structures:*

$$4_D\ 10_A\ 2_A\ 2_A\ 2_A\,, \quad 10_A\ 2_A\ 2_A\ 2_A\ 1_A\ 1_A\,.$$

For definiteness, we will focus on the last of these. Following the procedure described in section 2, we obtain the modified superpotential

$$\widetilde{W} = x_1^{12} + x_2^4 + x_3^4 + x_4^4 + x_5^3 + x_6^3 + z^2 + \eta_1\eta_2\,.$$

Employing the heuristic reasoning of [4], we find for $(n_{\eta_1}, n_{\eta_2}) = (3,9)$ *that the Hodge diamond of* $\mathcal{M}$ *is*

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$$
\begin{array}{ccccccc} &&&1&&& \\ &&0&&0&& \\ &0&&10&&0& \\ 1&&46&&46&&1 \\ &0&&10&&0& \\ &&0&&0&& \\ &&&1&&& \end{array}
=
\begin{array}{ccccccc} &&&1&&& \\ &&0&&0&& \\ &0&&0&&0& \\ 1&&46&&46&&1 \\ &0&&0&&0& \\ &&0&&0&& \\ &&&1&&& \end{array}
+
\begin{array}{ccccccc} &&&0&&& \\ &&0&&0&& \\ &0&&2&&0& \\ 0&&0&&0&&0 \\ &0&&2&&0& \\ &&0&&0&& \\ &&&0&&& \end{array}
+
\begin{array}{ccccccc} &&&0&&& \\ &&0&&0&& \\ &0&&1&&0& \\ 0&&0&&0&&0 \\ &0&&1&&0& \\ &&0&&0&& \\ &&&0&&& \end{array}
$$

$$
+
\begin{array}{ccccccc} &&&0&&& \\ &&0&&0&& \\ &0&&7&&0& \\ 0&&0&&0&&0 \\ &0&&7&&0& \\ &&0&&0&& \\ &&&0&&& \end{array} . \tag{4.4}
$$

The first three terms on the right-hand side of (4.4) have the same origin as the corresponding terms in example 4.2. The fourth term includes the contribution of the fixed point set associated with the fourth twisted sector (the lower 7) and the fixed point set associated with the ninth twisted sector (the upper 7). Our result (4.4) agrees with the Hodge diamond of the associated Landau-Ginzburg orbifold. For all other fermionic weight assigments consistent with (1.9), we do not obtain this agreement. Thus, in this example, using these heuristic arguments, we obtain a unique result

$$(n_{\eta_1}, n_{\eta_2}) = (3, 9)\,.$$

In contrast, conjecture 1.1 allows

$$(n_{\eta_1}, n_{\eta_2}) \in \{(3, 9), (6, 6)\}$$

and, as indicated by hypersurface family 8 of the appendix, the singular locus constraint (1.8) allows

$$(n_{\eta_1}, n_{\eta_2}) \in \{(1, 11), (2, 10), (3, 9), (4, 8), (5, 7), (6, 6)\}\,.$$

Example 4.4 *A hypersurface in the family* $\mathbf{WSP}(2, 3, 1, 2, 2, 2, 4|n_{\eta_1}, n_{\eta_2})[8]$ *can be obtained from a Gepner model with level/invariant structure*

$$6_D\ 6_A\ 2_A\ 2_A\ 2_A\,.$$

Following the procedure described in section 2, we obtain the modified superpotential

$$\widetilde{W} = x_1^4 + x_1 y_1^2 + x_2^8 + x_3^4 + x_4^4 + x_5^4 + z^2 + \eta_1\eta_2\,.$$

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Employing the heuristic reasoning of [4], we find for $(n_{\eta_1}, n_{\eta_2}) = (4,4)$ *that the Hodge diamond of* $\mathcal{M}$ *is*

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 73 & & 73 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 0 & & 0 & \\ 1 & & 63 & & 63 & & 1 \\ & 0 & & 0 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} + \begin{array}{ccccccc} & & & 0 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 0 & & 10 & & 10 & & 0 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 0 & & & \end{array}. \tag{4.5}$$

The first term on the right-hand side of (4.5) is the contribution arising from the untwisted sector whereas the second term arises from the fourth twisted sector. Our result (4.5) agrees with the Hodge diamond of the associated Landau-Ginzburg orbifold. For all other fermionic weight assigments consistent with (1.9), we do not obtain this agreement. Thus, in this example, using these heuristic arguments, we obtain a unique result

$$(n_{\eta_1}, n_{\eta_2}) = (4,4)\,.$$

In contrast, conjecture 1.1 and, as indicated by hypersurface family 3 of the appendix, the singular locus constraint (1.8) both allow

$$(n_{\eta_1}, n_{\eta_2}) \in \{(2,6),(4,4)\}\,.$$

Example 4.5 *Let us revisit the family of hypersurfaces discussed in example 3.1,* $\mathbf{WSP}(2,5,2,5,4,4,4,4,6|n_{\eta_1},n_{\eta_2},n_{\eta_3},n_{\eta_4})[12]$. *We again focus on the hypersurface obtained from a Gepner model with level/invariant structure* $10_D\ 10_D\ 1_A\ 1_A\ 1_A\ 1_A$. *Proceeding as in the above examples, we find for* $(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4}) = (2,10,4,8)$ *that the Hodge diamond of* $\mathcal{M}$ *is*

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 7 & & 0 & \\ 1 & & 79 & & 79 & & 1 \\ & 0 & & 7 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 0 & & 0 & \\ 1 & & 47 & & 47 & & 1 \\ & 0 & & 0 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} + \begin{array}{ccccccc} & & & 0 & & & \\ & & 0 & & 0 & & \\ & 0 & & 6 & & 0 & \\ 0 & & 0 & & 0 & & 0 \\ & 0 & & 6 & & 0 & \\ & & 0 & & 0 & & \\ & & & 0 & & & \end{array} + \begin{array}{ccccccc} & & & 0 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 0 & & 32 & & 32 & & 0 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 0 & & & \end{array}. \tag{4.6}$$

The terms on the right-hand side of (4.6) originate from the same sectors as the corresponding terms in example 4.2. Our result (4.6) agrees with the Hodge diamond of the associated Landau-Ginzburg orbifold. For all other fermionic weight assigments consistent with (1.9), we do not obtain this agreement. Thus, in this example, using these heuristic arguments, we obtain

$$(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4}) \in \{(2,10,4,8),(4,8,6,6)\}\,.$$

In contrast, conjecture 1.1 allows

$$(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4}) \in \{(2,10,4,8),(4,8,4,8),(4,8,6,6)\}$$

and the singular locus constraint (1.8) allows

$$\begin{aligned}(n_{\eta_1}, n_{\eta_2}, n_{\eta_3}, n_{\eta_4}) \in \{&(2,10,2,10),(2,10,4,8),(2,10,6,6),\\ &(4,8,4,8),(4,8,6,6),(6,6,6,6)\}\,.\end{aligned}$$

It is interesting to note that, in the above examples, the solutions obtained from the heuristic approach agree precisely with those obtained from conjecture 1.1 supplemented by the constraint

$$\widetilde{\mathcal{D}}_p \geq 0 \quad \text{whenever} \quad \mathcal{D}_p \geq 0\,. \tag{4.7}$$

Let us now consider an example in which the heuristic approach yields no solution.

Example 4.6 *A hypersurface in the family* $\mathbf{WSP}(1,1,1,2,2,2,3|n_{\eta_1}, n_{\eta_2})[6]$ *can be obtained from a Gepner model with either of the following level/invariant structures:*

$$4_D\ 4_A\ 4_A\ 4_A\ 1_A\,,\quad 4_A\ 4_A\ 4_A\ 1_A\ 1_A\ 1_A\,.$$

For definiteness, we will focus on the last of these. Following the procedure described in section 2, we obtain the modified superpotential

$$\widetilde{W} = x_1^6 + x_2^6 + x_3^6 + x_4^3 + x_5^3 + x_6^3 + z^2 + \eta_1\eta_2\,.$$

Employing the heuristic reasoning of [4], we find for $(n_{\eta_1}, n_{\eta_2}) \in \{(1,5),(3,3)\}$ *that the Hodge diamond of* $\mathcal{M}$ *is*

$$\begin{array}{ccccccc} &&&1&&&\\ &&0&&0&&\\ &0&&1&&0&\\ 1&&84&&84&&1\\ &0&&1&&0&\\ &&0&&0&&\\ &&&1&&& \end{array} = \begin{array}{ccccccc} &&&1&&&\\ &&0&&0&&\\ &0&&0&&0&\\ 1&&83&&83&&1\\ &0&&0&&0&\\ &&0&&0&&\\ &&&1&&& \end{array} + \begin{array}{ccccccc} &&&0&&&\\ &&0&&0&&\\ &0&&1&&0&\\ 0&&1&&1&&0\\ &0&&1&&0&\\ &&0&&0&&\\ &&&0&&& \end{array}. \tag{4.8}$$

The first term on the right-hand side of (4.8) is the contribution arising from the untwisted sector. The second term includes the contribution of the fixed point set associated with the third twisted sector. For $(n_{\eta_1}, n_{\eta_2}) = (2,4)$*, we find that the only contribution to the Hodge diamond of* $\mathcal{M}$ *arises from the untwisted sector. In all cases, the result disagrees with the Landau-Ginzburg orbifold Hodge diamond*

$$\begin{array}{ccccccc} &&&1&&&\\ &&0&&0&&\\ &0&&0&&0&\\ 1&&84&&84&&1\\ &0&&0&&0&\\ &&0&&0&&\\ &&&1&&& \end{array}. \tag{4.9}$$

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Thus, in this example, using these heuristic arguments, we find no solution for the fermionic weights. In contrast, conjecture 1.1 and, as indicated by hypersurface family 2 of the appendix, the singular locus constraint (1.8) both allow $(n_{\eta_1}, n_{\eta_2}) \in \{(1,5),(2,4),(3,3)\}$.

5 Conclusion

The analysis in section 4 compares the fermionic weights obtained from our computer program with those obtained by requiring agreement between the Hodge diamond of the Landau-Ginzburg orbifold and the heuristically determined Hodge diamond of the supervariety. Running the program with the singular locus constraint (1.8) yields at least one solution for each $\hat{c} = 3$ Gepner model associated with a supervariety through Sethi's proposed correspondence. Conjecture 1.1 is a stronger constraint, but it still yields at least one solution for the vast majority of these models. The heuristic approach places the strongest constraint on the fermionic weights. It yields a unique solution in examples 4.1, 4.2, 4.3, and 4.4, two solutions in example 4.5, and no solution in example 4.6. In the examples we have studied, the heuristically determined solutions are a subset of the solutions obtained from conjecture 1.1. Furthermore, in these examples, when the heuristic approach yields solutions, these solutions agree precisely with those obtained from conjecture 1.1 supplemented by the constraint (4.7). Thus, something seems to be "right" about the combination of conjecture 1.1 and the constraint (4.7). A proper supercohomology theory would allow more conclusive statements to be made.

In the appendix, table 2 indicates when the Newton polytope of $\widetilde{W}_{bos}$ admits a nef partition. In this case, the Landau-Ginzburg orbifold can be given a geometrical interpretation as a nonlinear sigma model on a complete intersection Calabi-Yau manifold. The complete intersection Calabi-Yau manifold should be equivalent to the Calabi-Yau supermanifold prescribed by Sethi's proposal. It can be shown that a reflexive Gorenstein polytope Δ admits a nef partition if and only if the reflexive Gorenstein cones σ_Δ and $\sigma_\Delta^\vee$ are both completely split [8]. In fact, the Landau-Ginzburg orbifold can be given a complete intersection Calabi-Yau manifold interpretation even when only σ_Δ is completely split [8]. In example 4.4, the Newton polytope of $\widetilde{W}_{bos}$ is nonreflexive Gorenstein. It turns out that, for all of the remaining examples in section 4, the Newton polytope of $\widetilde{W}_{bos}$ is reflexive Gorenstein but σ_Δ is not completely split. Thus, in these examples, the Landau-Ginzburg orbifold cannot be given a complete intersection Calabi-Yau manifold interpretation. We leave a detailed investigation of the cases in which only σ_Δ or only $\sigma_\Delta^\vee$ is completely split to future work.

A Supervariety hypersurface families

In table 2, we list the supervariety hypersurface families associated with $\hat{c} = 3$ Gepner models. A hypersurface family corresponding to a hypersurface $\widetilde{W} = 0$ which defines a supervariety $\mathcal{M}$ embedded in $\mathbf{WSP}(n_{z_1}, \ldots, n_{z_{n+1}} | n_{\eta_1}, \ldots, n_{\eta_{2m}})$ is denoted by $\mathbf{WSP}(n_{z_1}, \ldots, n_{z_{n+1}} | n_{\eta_1}, \ldots, n_{\eta_{2m}})[d]$. Here, $\widetilde{W}$ is the modified superpotential obtained by satisfying (2.8) and d is its degree of quasihomogeneity. The fermionic weights are determined by requiring (1.8) and (1.9) to be satisfied. The solutions for these fermionic weights are parameterized by $k = 1, \ldots, [\frac{d}{2s_k}]$ and $l = 1, \ldots, [\frac{d}{2s_l}]$, where s_k and s_l are the coefficients of k and l in the first and third fermion weight assignments, respectively. For each hypersurface family, the Hodge numbers $h^{1,1}$ and $h^{2,1}$ and the Euler number $\chi = 2\left(h^{1,1} - h^{2,1}\right)$ of the associated Landau-Ginzburg orbifold are given. When the Newton polytope of $\widetilde{W}_{bos}$ admits a nef partition, this is indicated by "nef". In a number of cases, the Landau-Ginzburg orbifold can be given a geometrical interpretation as a product of a two-torus and a K3 surface, which is indicated by "$T^2 \times K3$". Finally, when the Newton polytope of $\widetilde{W}_{bos}$ is nonreflexive Gorenstein, this is indicated by "nonRG".

#	hypersurface family	$h^{1,1}$	$h^{2,1}$	χ	Comments
1	$\mathbf{WSP}(1,1,1,1,1,1,2\|k,4-k)[4]$	0	90	-180	
2	$\mathbf{WSP}(1,1,1,2,2,2,3\|k,6-k)[6]$	0	84	-168	
3	$\mathbf{WSP}(2,3,1,2,2,2,4\|2k,8-2k)[8]$	1	73	-144	nonRG
4	$\mathbf{WSP}(2,3,2,3,1,1,4\|k,8-k)[8]$	1	77	-152	
5	$\mathbf{WSP}(2,4,2,4,1,2,5\|2k,10-2k)[10]$	1	85	-168	
6	$\mathbf{WSP}(2,3,3,3,3,4,6\|3k,12-3k)[12]$	1	61	-120	
7	$\mathbf{WSP}(2,2,3,3,4,4,6\|2k,12-2k)[12]$	3	51	-96	
8	$\mathbf{WSP}(1,3,3,3,4,4,6\|k,12-k)[12]$	10	46	-72	
9	$\mathbf{WSP}(1,2,3,4,4,4,6\|2k,12-2k)[12]$	2	62	-120	
10	$\mathbf{WSP}(1,1,4,4,4,4,6\|2k,12-2k)[12]$	7	79	-144	
11	$\mathbf{WSP}(2,5,2,3,3,3,6\|k,12-k)[12]$	9	39	-60	
12	$\mathbf{WSP}(2,5,2,2,3,4,6\|2k,12-2k)[12]$	2	74	-144	nonRG
13	$\mathbf{WSP}(2,5,1,3,3,4,6\|k,12-k)[12]$	1	61	-120	nonRG
14	$\mathbf{WSP}(2,5,1,2,4,4,6\|2k,12-2k)[12]$	3	75	-144	nonRG
15	$\mathbf{WSP}(2,5,2,5,2,2,6\|2k,12-2k)[12]$	2	128	-252	
16	$\mathbf{WSP}(2,5,2,5,1,3,6\|k,12-k)[12]$	3	69	-132	
17	$\mathbf{WSP}(4,6,2,7,1,4,8\|2k,16-2k)[16]$	3	75	-144	
18	$\mathbf{WSP}(2,7,2,7,2,4,8\|2k,16-2k)[16]$	4	148	-288	nef
19	$\mathbf{WSP}(6,4,2,3,6,6,9\|6,12)[18]$	2	62	-120	nonRG
20	$\mathbf{WSP}(6,4,6,4,1,6,9\|2k,18-2k)[18]$	2	56	-108	
21	$\mathbf{WSP}(1,2,6,6,6,6,9\|6,12)[18]$	8	68	-120	
22	$\mathbf{WSP}(2,8,2,3,6,6,9\|2k,18-2k)[18]$	8	68	-120	
23	$\mathbf{WSP}(6,4,2,8,1,6,9\|2k,18-2k)[18]$	4	76	-144	
24	$\mathbf{WSP}(2,8,2,8,1,6,9\|2k,18-2k)[18]$	2	110	-216	
25	$\mathbf{WSP}(4,8,4,4,5,5,10\|2k,20-2k)[20]$	21	21	0	nef, $T^2 \times K3$
26	$\mathbf{WSP}(4,8,4,8,1,5,10\|2k,20-2k)[20]$	13	49	-72	
27	$\mathbf{WSP}(2,9,4,5,5,5,10\|k,20-k)[20]$	17	29	-24	
28	$\mathbf{WSP}(4,8,2,9,2,5,10\|2k,20-2k)[20]$	7	79	-144	nonRG
29	$\mathbf{WSP}(2,9,2,9,4,4,10\|2k,20-2k)[20]$	7	143	-272	nef
30	$\mathbf{WSP}(3,3,6,8,8,8,12\|2k,24-2k)[24]$	21	21	0	nef, $T^2 \times K3$
31	$\mathbf{WSP}(1,3,8,8,8,8,12\|4k,24-4k)[24]$	16	52	-72	
32	$\mathbf{WSP}(6,9,3,4,6,8,12\|6k,24-6k)[24]$	3	51	-96	
33	$\mathbf{WSP}(6,9,1,6,6,8,12\|6k,24-6k)[24]$	10	46	-72	nonRG
34	$\mathbf{WSP}(6,9,2,3,8,8,12\|2k,24-2k)[24]$	13	37	-48	nonRG
35	$\mathbf{WSP}(6,9,1,4,8,8,12\|2k,24-2k)[24]$	12	48	-72	nonRG
36	$\mathbf{WSP}(6,9,6,9,2,4,12\|6k,24-6k)[24]$	6	66	-120	nef
37	$\mathbf{WSP}(4,10,3,3,8,8,12\|2k,24-2k)[24]$	11	35	-48	
38	$\mathbf{WSP}(6,9,4,10,3,4,12\|2k,24-2k)[24]$	11	35	-48	
39	$\mathbf{WSP}(6,9,4,10,1,6,12\|2k,24-2k)[24]$	7	55	-96	nonRG
40	$\mathbf{WSP}(2,11,3,6,6,8,12\|2k,24-2k)[24]$	10	46	-72	
41	$\mathbf{WSP}(2,11,3,4,8,8,12\|2k,24-2k)[24]$	12	48	-72	nonRG
42	$\mathbf{WSP}(2,11,1,6,8,8,12\|2k,24-2k)[24]$	9	81	-144	
43	$\mathbf{WSP}(6,9,2,11,4,4,12\|2k,24-2k)[24]$	6	90	-168	nef
44	$\mathbf{WSP}(6,9,2,11,2,6,12\|2k,24-2k)[24]$	6	114	-216	nef
45	$\mathbf{WSP}(4,10,2,11,3,6,12\|2k,24-2k)[24]$	7	55	-96	nonRG
46	$\mathbf{WSP}(4,10,2,11,1,8,12\|2k,24-2k)[24]$	3	99	-192	

47	$\mathbf{WSP}(2,11,2,11,4,6,12\vert 2k,24-2k)[24]$	8	164	-312	nef
48	$\mathbf{WSP}(2,11,2,11,2,8,12\vert 2k,24-2k)[24]$	3	243	-480	nef
49	$\mathbf{WSP}(4,12,2,13,4,7,14\vert 2k,28-2k)[28]$	8	80	-144	
50	$\mathbf{WSP}(3,6,6,10,10,10,15\vert 2k,30-2k)[30]$	21	21	0	nef, $T^2 \times K3$
51	$\mathbf{WSP}(2,3,10,10,10,10,15\vert 10,20)[30]$	17	41	-48	
52	$\mathbf{WSP}(6,12,5,6,6,10,15\vert 6k,30-6k)[30]$	21	21	0	nef, $T^2 \times K3$
53	$\mathbf{WSP}(6,12,2,5,10,10,15\vert 2k,30-2k)[30]$	17	41	-48	
54	$\mathbf{WSP}(6,12,1,6,10,10,15\vert 2k,30-2k)[30]$	7	55	-96	nonRG
55	$\mathbf{WSP}(2,14,3,6,10,10,15\vert 2k,30-2k)[30]$	7	55	-96	
56	$\mathbf{WSP}(6,12,2,14,5,6,15\vert 2k,30-2k)[30]$	7	55	-96	nonRG
57	$\mathbf{WSP}(6,12,2,14,1,10,15\vert 2k,30-2k)[30]$	5	101	-192	
58	$\mathbf{WSP}(2,14,2,14,3,10,15\vert 2k,30-2k)[30]$	5	101	-192	
59	$\mathbf{WSP}(12,8,4,9,9,12,18\vert 6k,36-6k)[36]$	21	21	0	nef, $T^2 \times K3$
60	$\mathbf{WSP}(12,8,1,9,12,12,18\vert 6k,36-6k)[36]$	10	46	-72	
61	$\mathbf{WSP}(12,8,6,15,4,9,18\vert 6k,36-6k)[36]$	8	44	-72	nonRG
62	$\mathbf{WSP}(12,8,6,15,1,12,18\vert 6k,36-6k)[36]$	13	49	-72	nonRG
63	$\mathbf{WSP}(4,16,4,9,9,12,18\vert 2k,36-2k)[36]$	21	21	0	nef, $T^2 \times K3$
64	$\mathbf{WSP}(4,16,1,9,12,12,18\vert 2k,36-2k)[36]$	20	56	-72	
65	$\mathbf{WSP}(6,15,4,16,4,9,18\vert 2k,36-2k)[36]$	14	50	-72	
66	$\mathbf{WSP}(6,15,4,16,1,12,18\vert 2k,36-2k)[36]$	5	77	-144	
67	$\mathbf{WSP}(12,8,2,17,6,9,18\vert 2k,36-2k)[36]$	11	53	-84	
68	$\mathbf{WSP}(12,8,2,17,3,12,18\vert 2k,36-2k)[36]$	13	49	-72	nonRG
69	$\mathbf{WSP}(2,17,2,9,12,12,18\vert 2k,36-2k)[36]$	16	100	-168	
70	$\mathbf{WSP}(6,15,2,17,2,12,18\vert 2k,36-2k)[36]$	5	185	-360	nef
71	$\mathbf{WSP}(4,16,2,17,6,9,18\vert 2k,36-2k)[36]$	13	73	-120	nonRG
72	$\mathbf{WSP}(4,16,2,17,3,12,18\vert 2k,36-2k)[36]$	5	77	-144	nonRG
73	$\mathbf{WSP}(2,17,2,17,4,12,18\vert 2k,36-2k)[36]$	7	271	-528	nef
74	$\mathbf{WSP}(10,15,10,15,2,8,20\vert 10k,40-10k)[40]$	7	63	-112	nef
75	$\mathbf{WSP}(10,15,8,16,1,10,20\vert 2k,40-2k)[40]$	13	49	-72	
76	$\mathbf{WSP}(10,15,4,18,5,8,20\vert 2k,40-2k)[40]$	19	27	-16	
77	$\mathbf{WSP}(10,15,2,19,4,10,20\vert 2k,40-2k)[40]$	12	96	-168	nef
78	$\mathbf{WSP}(8,16,2,19,5,10,20\vert 2k,40-2k)[40]$	13	49	-72	
79	$\mathbf{WSP}(2,19,2,19,8,10,20\vert 2k,40-2k)[40]$	11	227	-432	nef
80	$\mathbf{WSP}(1,6,14,14,14,14,21\vert 14,28)[42]$	23	47	-48	
81	$\mathbf{WSP}(6,18,6,18,1,14,21\vert 6k,42-6k)[42]$	15	63	-96	
82	$\mathbf{WSP}(2,20,6,7,14,14,21\vert 2k,42-2k)[42]$	23	47	-48	
83	$\mathbf{WSP}(6,18,2,20,3,14,21\vert 2k,42-2k)[42]$	15	63	-96	
84	$\mathbf{WSP}(6,21,1,12,16,16,24\vert 2k,48-2k)[48]$	20	56	-72	
85	$\mathbf{WSP}(8,20,6,21,1,16,24\vert 2k,48-2k)[48]$	8	68	-120	
86	$\mathbf{WSP}(6,21,6,21,2,16,24\vert 6k,48-6k)[48]$	7	127	-240	nef
87	$\mathbf{WSP}(6,21,4,22,3,16,24\vert 2k,48-2k)[48]$	17	41	-48	
88	$\mathbf{WSP}(2,23,3,12,16,16,24\vert 2k,48-2k)[48]$	20	56	-72	
89	$\mathbf{WSP}(12,18,2,23,1,16,24\vert 2k,48-2k)[48]$	9	129	-240	
90	$\mathbf{WSP}(8,20,2,23,3,16,24\vert 2k,48-2k)[48]$	8	68	-120	nonRG
91	$\mathbf{WSP}(6,21,2,23,8,12,24\vert 2k,48-2k)[48]$	16	112	-192	
92	$\mathbf{WSP}(6,21,2,23,4,16,24\vert 2k,48-2k)[48]$	8	164	-312	nonRG
93	$\mathbf{WSP}(2,23,2,23,6,16,24\vert 2k,48-2k)[48]$	9	321	-624	nef
94	$\mathbf{WSP}(3,12,15,20,20,20,30\vert 10k,60-10k)[60]$	21	21	0	nef, $T^2 \times K3$
95	$\mathbf{WSP}(12,24,4,15,15,20,30\vert 6k,60-6k)[60]$	21	21	0	nef, $T^2 \times K3$
96	$\mathbf{WSP}(10,25,3,12,20,20,30\vert 10k,60-10k)[60]$	23	23	0	
97	$\mathbf{WSP}(12,24,10,25,4,15,30\vert 2k,60-2k)[60]$	29	29	0	
98	$\mathbf{WSP}(6,27,10,12,15,20,30\vert 6k,60-6k)[60]$	15	39	-48	
99	$\mathbf{WSP}(6,27,5,12,20,20,30\vert 2k,60-2k)[60]$	23	23	0	
100	$\mathbf{WSP}(6,27,2,15,20,20,30\vert 2k,60-2k)[60]$	31	55	-48	
101	$\mathbf{WSP}(12,24,6,27,1,20,30\vert 6k,60-6k)[60]$	17	65	-96	
102	$\mathbf{WSP}(10,25,6,27,10,12,30\vert 2k,60-2k)[60]$	13	49	-72	
103	$\mathbf{WSP}(10,25,6,27,2,20,30\vert 2k,60-2k)[60]$	10	106	-192	nonRG
104	$\mathbf{WSP}(6,27,6,27,4,20,30\vert 6k,60-6k)[60]$	11	107	-192	nef
105	$\mathbf{WSP}(4,28,3,15,20,20,30\vert 2k,60-2k)[60]$	31	31	0	
106	$\mathbf{WSP}(10,25,4,28,3,20,30\vert 2k,60-2k)[60]$	10	46	-72	nonRG
107	$\mathbf{WSP}(6,27,4,28,10,15,30\vert 2k,60-2k)[60]$	25	37	-24	nonRG

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108	$\mathbf{WSP}(6,27,4,28,5,20,30\|2k,60-2k)[60]$	10	46	-72	nonRG
109	$\mathbf{WSP}(2,29,12,12,15,20,30\|2k,60-2k)[60]$	23	47	-48	
110	$\mathbf{WSP}(2,29,4,15,20,20,30\|2k,60-2k)[60]$	26	86	-120	
111	$\mathbf{WSP}(12,24,2,29,3,20,30\|2k,60-2k)[60]$	17	65	-96	
112	$\mathbf{WSP}(10,25,2,29,12,12,30\|2k,60-2k)[60]$	25	85	-120	
113	$\mathbf{WSP}(10,25,2,29,4,20,30\|2k,60-2k)[60]$	11	155	-188	nef
114	$\mathbf{WSP}(6,27,2,29,6,20,30\|2k,60-2k)[60]$	10	178	-336	
115	$\mathbf{WSP}(4,28,2,29,12,15,30\|2k,60-2k)[60]$	17	101	-168	
116	$\mathbf{WSP}(18,27,4,34,1,24,36\|2k,72-2k)[72]$	14	98	-168	
117	$\mathbf{WSP}(18,27,2,35,8,18,36\|2k,72-2k)[72]$	19	91	-144	nef
118	$\mathbf{WSP}(18,27,2,35,2,24,36\|2k,72-2k)[72]$	14	242	-456	
119	$\mathbf{WSP}(6,33,2,35,8,24,36\|2k,72-2k)[72]$	15	183	-336	
120	$\mathbf{WSP}(4,34,2,35,9,24,36\|2k,72-2k)[72]$	14	98	-168	nonRG
121	$\mathbf{WSP}(12,36,1,21,28,28,42\|2k,84-2k)[84]$	45	45	0	
122	$\mathbf{WSP}(14,35,12,36,1,28,42\|2k,84-2k)[84]$	15	63	-96	
123	$\mathbf{WSP}(6,39,4,21,28,28,42\|2k,84-2k)[84]$	41	41	0	
124	$\mathbf{WSP}(14,35,6,39,4,28,42\|2k,84-2k)[84]$	16	76	-120	nonRG
125	$\mathbf{WSP}(4,40,12,21,21,28,42\|2k,84-2k)[84]$	21	21	0	nef, $T^2\times K3$
126	$\mathbf{WSP}(14,35,4,40,12,21,42\|2k,84-2k)[84]$	35	35	0	
127	$\mathbf{WSP}(2,41,6,21,28,28,42\|2k,84-2k)[84]$	40	76	-72	
128	$\mathbf{WSP}(14,35,2,41,6,28,42\|2k,84-2k)[84]$	15	147	-264	nef
129	$\mathbf{WSP}(12,36,2,41,14,21,42\|2k,84-2k)[84]$	34	58	-48	
130	$\mathbf{WSP}(12,36,2,41,7,28,42\|2k,84-2k)[84]$	15	63	-96	
131	$\mathbf{WSP}(2,41,2,41,12,28,42\|2k,84-2k)[84]$	11	491	-960	nef
132	$\mathbf{WSP}(30,20,18,36,1,30,45\|6k,90-6k)[90]$	29	41	-24	nonRG
133	$\mathbf{WSP}(18,36,10,40,1,30,45\|2k,90-2k)[90]$	17	65	-96	
134	$\mathbf{WSP}(30,20,2,44,9,30,45\|2k,90-2k)[90]$	29	41	-24	nonRG
135	$\mathbf{WSP}(18,36,2,44,5,30,45\|2k,90-2k)[90]$	17	65	-96	
136	$\mathbf{WSP}(10,40,2,44,9,30,45\|2k,90-2k)[90]$	17	65	-96	
137	$\mathbf{WSP}(24,36,6,45,1,32,48\|6k,96-6k)[96]$	24	84	-120	
138	$\mathbf{WSP}(24,36,2,47,3,32,48\|2k,96-2k)[96]$	24	84	-120	
139	$\mathbf{WSP}(6,45,2,47,12,32,48\|2k,96-2k)[96]$	18	222	-408	
140	$\mathbf{WSP}(30,45,1,24,40,40,60\|10k,120-10k)[120]$	39	39	0	nonRG
141	$\mathbf{WSP}(24,48,10,55,3,40,60\|2k,120-2k)[120]$	29	29	0	
142	$\mathbf{WSP}(30,45,8,56,1,40,60\|2k,120-2k)[120]$	24	84	-120	
143	$\mathbf{WSP}(30,45,6,57,2,40,60\|6k,120-6k)[120]$	23	143	-240	
144	$\mathbf{WSP}(24,48,6,57,5,40,60\|6k,120-6k)[120]$	29	29	0	
145	$\mathbf{WSP}(10,55,6,57,12,40,60\|2k,120-2k)[120]$	22	82	-120	
146	$\mathbf{WSP}(30,45,4,58,3,40,60\|2k,120-2k)[120]$	29	53	-48	
147	$\mathbf{WSP}(6,57,4,58,15,40,60\|2k,120-2k)[120]$	29	53	-48	nonRG
148	$\mathbf{WSP}(2,59,15,24,40,40,60\|2k,120-2k)[120]$	39	39	0	
149	$\mathbf{WSP}(30,45,2,59,20,24,60\|2k,120-2k)[120]$	33	69	-72	
150	$\mathbf{WSP}(30,45,2,59,4,40,60\|2k,120-2k)[120]$	24	204	-360	
151	$\mathbf{WSP}(10,55,2,59,24,30,60\|2k,120-2k)[120]$	33	141	-216	
152	$\mathbf{WSP}(8,56,2,59,15,40,60\|2k,120-2k)[120]$	24	84	-120	
153	$\mathbf{WSP}(10,65,4,68,28,35,70\|2k,140-2k)[140]$	47	47	0	
154	$\mathbf{WSP}(28,56,2,69,20,35,70\|2k,140-2k)[140]$	53	53	0	
155	$\mathbf{WSP}(2,77,12,39,52,52,78\|2k,156-2k)[156]$	71	71	0	
156	$\mathbf{WSP}(26,65,2,77,12,52,78\|2k,156-2k)[156]$	23	143	-240	nef
157	$\mathbf{WSP}(42,63,12,78,1,56,84\|6k,168-6k)[168]$	38	74	-72	
158	$\mathbf{WSP}(42,63,8,80,3,56,84\|2k,168-2k)[168]$	39	39	0	
159	$\mathbf{WSP}(42,63,6,81,4,56,84\|6k,168-6k)[168]$	33	105	-144	
160	$\mathbf{WSP}(8,80,6,81,21,56,84\|2k,168-2k)[168]$	39	39	0	
161	$\mathbf{WSP}(42,63,2,83,6,56,84\|2k,168-2k)[168]$	34	190	-312	
162	$\mathbf{WSP}(12,78,2,83,21,56,84\|2k,168-2k)[168]$	38	74	-72	
163	$\mathbf{WSP}(6,81,2,83,24,56,84\|2k,168-2k)[168]$	23	335	-624	
164	$\mathbf{WSP}(18,81,2,89,20,60,90\|2k,180-2k)[180]$	42	150	-216	
165	$\mathbf{WSP}(54,81,2,107,8,72,108\|2k,216-2k)[216]$	48	180	-264	
166	$\mathbf{WSP}(20,100,2,109,44,55,110\|2k,220-2k)[220]$	71	71	0	
167	$\mathbf{WSP}(48,96,30,105,1,80,120\|6k,240-6k)[240]$	53	53	0	
168	$\mathbf{WSP}(48,96,2,119,15,80,120\|2k,240-2k)[240]$	53	53	0	

169	$\mathbf{WSP}(30,105,2,119,24,80,120\|2k,240-2k)[240]$	50	134	-168	
170	$\mathbf{WSP}(66,99,6,129,8,88,132\|6k,264-6k)[264]$	57	81	-48	
171	$\mathbf{WSP}(78,117,24,144,1,104,156\|6k,312-6k)[312]$	69	69	0	
172	$\mathbf{WSP}(78,117,2,155,12,104,156\|2k,312-2k)[312]$	66	174	-216	
173	$\mathbf{WSP}(24,144,2,155,39,104,156\|2k,312-2k)[312]$	69	69	0	
174	$\mathbf{WSP}(14,161,2,167,48,112,168\|2k,336-2k)[336]$	47	287	-480	
175	$\mathbf{WSP}(14,203,6,207,60,140,210\|2k,420-2k)[420]$	59	131	-144	
176	$\mathbf{WSP}(150,225,2,299,24,200,300\|2k,600-2k)[600]$	119	167	-96	
177	$\mathbf{WSP}(42,441,2,461,132,308,462\|2k,924-2k)[924]$	137	257	-240	
178	$\mathbf{WSP}(1,2,1,2,1,2,1\|k,5-k)[5]$	1	85	-168	
179	$\mathbf{WSP}(1,1,2,2,2,2,2\|2,4)[6]$	1	73	-144	
180	$\mathbf{WSP}(2,3,2,3,2,2,2\|2k,8-2k)[8]$	2	86	-168	nef
181	$\mathbf{WSP}(2,3,2,3,2,3,1\|k,8-k)[8]$	2	58	-112	
182	$\mathbf{WSP}(3,2,3,2,3,2,3\|k,9-k)[9]$	8	35	-54	nef
183	$\mathbf{WSP}(3,2,1,3,3,3,3\|3,6)[9]$	2	62	-120	nonRG
184	$\mathbf{WSP}(3,2,3,2,1,4,3\|k,9-k)[9]$	2	56	-108	
185	$\mathbf{WSP}(1,4,1,3,3,3,3\|k,9-k)[9]$	8	68	-120	
186	$\mathbf{WSP}(3,2,1,4,1,4,3\|k,9-k)[9]$	4	76	-144	
187	$\mathbf{WSP}(1,4,1,4,1,4,3\|k,9-k)[9]$	2	110	-216	
188	$\mathbf{WSP}(3,3,3,3,4,4,4\|k,12-k)[12]$	21	21	0	nef, $T^2\times K3$
189	$\mathbf{WSP}(2,3,3,4,4,4,4\|2k,12-2k)[12]$	11	35	-48	
190	$\mathbf{WSP}(1,3,4,4,4,4,4\|4,8)[12]$	2	62	-120	
191	$\mathbf{WSP}(2,5,3,3,3,4,4\|k,12-k)[12]$	4	40	-72	
192	$\mathbf{WSP}(2,5,2,3,4,4,4\|2k,12-2k)[12]$	2	62	-120	nonRG
193	$\mathbf{WSP}(2,5,1,4,4,4,4\|2k,12-2k)[12]$	9	57	-96	
194	$\mathbf{WSP}(2,5,2,5,3,3,4\|k,12-k)[12]$	5	41	-72	
195	$\mathbf{WSP}(2,5,2,5,2,4,4\|2k,12-2k)[12]$	5	101	-192	nef
196	$\mathbf{WSP}(2,5,2,5,2,5,3\|k,12-k)[12]$	3	57	-108	
197	$\mathbf{WSP}(3,6,3,3,5,5,5\|k,15-k)[15]$	21	21	0	nef, $T^2\times K3$
198	$\mathbf{WSP}(3,6,1,5,5,5,5\|k,15-k)[15]$	17	41	-48	
199	$\mathbf{WSP}(3,6,1,7,3,5,5\|k,15-k)[15]$	7	55	-96	nonRG
200	$\mathbf{WSP}(3,6,1,7,1,7,5\|k,15-k)[15]$	5	101	-192	
201	$\mathbf{WSP}(4,6,2,7,2,7,4\|2k,16-2k)[16]$	8	104	-192	nef
202	$\mathbf{WSP}(4,8,4,8,2,9,5\|2k,20-2k)[20]$	13	49	-72	
203	$\mathbf{WSP}(1,10,3,7,7,7,7\|k,21-k)[21]$	23	47	-48	
204	$\mathbf{WSP}(3,9,3,9,1,10,7\|k,21-k)[21]$	15	63	-96	
205	$\mathbf{WSP}(6,9,3,6,8,8,8\|2k,24-2k)[24]$	21	21	0	nef, $T^2\times K3$
206	$\mathbf{WSP}(6,9,1,8,8,8,8\|2k,24-2k)[24]$	24	36	-24	
207	$\mathbf{WSP}(6,9,6,9,4,6,8\|6k,24-6k)[24]$	7	55	-96	nef
208	$\mathbf{WSP}(6,9,6,9,2,8,8\|2k,24-2k)[24]$	19	43	-48	nef
209	$\mathbf{WSP}(6,9,4,10,3,8,8\|2k,24-2k)[24]$	9	33	-48	
210	$\mathbf{WSP}(6,9,6,9,4,10,4\|2k,24-2k)[24]$	20	32	-24	nef
211	$\mathbf{WSP}(2,11,3,8,8,8,8\|2k,24-2k)[24]$	24	36	-24	
212	$\mathbf{WSP}(6,9,2,11,6,6,8\|2k,24-2k)[24]$	8	68	-120	nonRG
213	$\mathbf{WSP}(6,9,2,11,4,8,8\|2k,24-2k)[24]$	10	70	-120	nonRG
214	$\mathbf{WSP}(6,9,4,10,2,11,6\|2k,24-2k)[24]$	10	70	-120	nonRG
215	$\mathbf{WSP}(2,11,2,11,6,8,8\|2k,24-2k)[24]$	11	131	-240	nef
216	$\mathbf{WSP}(4,10,2,11,2,11,8\|2k,24-2k)[24]$	9	153	-288	nef
217	$\mathbf{WSP}(12,8,2,17,9,12,12\|2k,36-2k)[36]$	10	46	-72	
218	$\mathbf{WSP}(12,8,6,15,2,17,12\|2k,36-2k)[36]$	17	77	-120	nonRG
219	$\mathbf{WSP}(4,16,2,17,9,12,12\|2k,36-2k)[36]$	20	56	-72	
220	$\mathbf{WSP}(6,15,4,16,2,17,12\|2k,36-2k)[36]$	13	109	-192	nef
221	$\mathbf{WSP}(10,15,10,15,4,18,8\|2k,40-2k)[40]$	31	23	16	nef
222	$\mathbf{WSP}(10,15,8,16,2,19,10\|2k,40-2k)[40]$	23	59	-72	
223	$\mathbf{WSP}(15,10,9,18,1,22,15\|k,45-k)[45]$	29	41	-24	nonRG
224	$\mathbf{WSP}(9,18,5,20,1,22,15\|k,45-k)[45]$	17	65	-96	
225	$\mathbf{WSP}(6,21,6,21,4,22,16\|2k,48-2k)[48]$	31	55	-48	nef
226	$\mathbf{WSP}(6,21,2,23,12,16,16\|2k,48-2k)[48]$	26	86	-120	
227	$\mathbf{WSP}(8,20,6,21,2,23,16\|2k,48-2k)[48]$	18	102	-168	
228	$\mathbf{WSP}(12,18,2,23,2,23,16\|2k,48-2k)[48]$	13	229	-432	nef
229	$\mathbf{WSP}(6,27,12,15,20,20,20\|2k,60-2k)[60]$	21	21	0	nef, $T^2\times K3$

230	$\mathbf{WSP}(10,25,6,27,12,20,20\|2k,60-2k)[60]$	31	31	0	
231	$\mathbf{WSP}(6,27,4,28,15,20,20\|2k,60-2k)[60]$	31	31	0	
232	$\mathbf{WSP}(10,25,6,27,4,28,20\|2k,60-2k)[60]$	22	58	-72	nonRG
233	$\mathbf{WSP}(12,24,6,27,2,29,20\|2k,60-2k)[60]$	21	117	-192	
234	$\mathbf{WSP}(18,27,4,34,2,35,24\|2k,72-2k)[72]$	22	130	-216	
235	$\mathbf{WSP}(12,36,2,41,21,28,28\|2k,84-2k)[84]$	45	45	0	
236	$\mathbf{WSP}(14,35,12,36,2,41,28\|2k,84-2k)[84]$	37	85	-96	
237	$\mathbf{WSP}(24,36,6,45,2,47,32\|2k,96-2k)[96]$	26	158	-264	
238	$\mathbf{WSP}(24,48,10,55,6,57,40\|2k,120-2k)[120]$	49	49	0	
239	$\mathbf{WSP}(30,45,6,57,4,58,40\|2k,120-2k)[120]$	43	67	-48	
240	$\mathbf{WSP}(30,45,2,59,24,40,40\|2k,120-2k)[120]$	55	55	0	
241	$\mathbf{WSP}(30,45,8,56,2,59,40\|2k,120-2k)[120]$	34	118	-168	
242	$\mathbf{WSP}(42,63,8,80,6,81,56\|2k,168-2k)[168]$	55	55	0	
243	$\mathbf{WSP}(42,63,12,78,2,83,56\|2k,168-2k)[168]$	46	106	-120	nonRG
244	$\mathbf{WSP}(48,96,30,105,2,119,80\|2k,240-2k)[240]$	89	89	0	
245	$\mathbf{WSP}(78,117,24,144,2,155,104\|2k,312-2k)[312]$	97	97	0	
246	$\mathbf{WSP}(1,2,2,2,2,2,2,2,3\|2,4,2,4)[6]$	0	84	-168	
247	$\mathbf{WSP}(2,3,2,3,2,3,2,3,4\|k,8-k,2,6)[8]$	1	53	-104	
	$\mathbf{WSP}(2,3,2,3,2,3,2,3,4\|3,5,4,4)[8]$	1	53	-104	
248	$\mathbf{WSP}(3,3,4,4,4,4,4,4,6\|2k,12-2k,4,8)[12]$	21	21	0	$T^2\times K3$
249	$\mathbf{WSP}(2,5,3,4,4,4,4,4,6\|2k,12-2k,4,8)[12]$	2	62	-120	nonRG
250	$\mathbf{WSP}(2,5,2,5,4,4,4,4,6\|2k,12-2k,2l,12-2l)[12]$	7	79	-144	
251	$\mathbf{WSP}(6,9,6,9,6,8,8,8,12\|2k,24-2k,6l,24-6l)[24]$	21	21	0	nef, $T^2\times K3$
252	$\mathbf{WSP}(6,9,6,9,4,10,8,8,12\|2k,24-2k,6l,24-6l)[24]$	11	35	-48	
253	$\mathbf{WSP}(6,9,2,11,8,8,8,8,12\|2k,24-2k,4l,24-4l)[24]$	16	52	-72	
254	$\mathbf{WSP}(1,1,1,1,1,1,1,1,1\|1,2,1,2)[3]$	0	84	-168	

Table 2: Supervariety hypersurface families associated with $\hat{c}=3$ Gepner models.

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